PROBLEM SET 3

Problem 1. *For $G = \mathsf{PGL}_2$, show that the fibers of $\mathsf{Bun}_B^{-d} \to \mathsf{Bun}_G$ is either empty or connected for $d >> 0$.*

Problem 2. *For $G = \mathsf{PGL}_2$ and $\check{G} = \mathsf{SL}_2$, consider the standard representation $\mathsf{std} \in \mathsf{Rep}(\check{G})$. Verify the Hecke functors*

$$\mathsf{H}_{\mathsf{std}} : \mathsf{Shv}(\mathsf{Bun}_G) \to \mathsf{Shv}(\mathsf{Bun}_G \times X)$$

defined in the lectures of Day 3 and Day 4 are canonically equivalent.

Problem 3. *For $x \in X$ and V_1, $V_2 \in \mathsf{Rep}(\check{G})$, construct a canonical equivalence*

$$\mathsf{H}_{V_1,x} \circ \mathsf{H}_{V_2,x} \simeq \mathsf{H}_{V_2,x} \circ \mathsf{H}_{V_1,x} \simeq \mathsf{H}_{V_1 \otimes V_2,x}$$

as endo-functors on $\mathsf{Shv}(\mathsf{Bun}_G)$.

Problem 4. *Show that any $\check{G}$-local system σ is uniquely determined by the symmetric monoidal functor*

$$\mathsf{ev}_\sigma : \mathsf{Rep}(\check{G})_{\mathsf{Ran}} \to \mathsf{Vect}.$$